SCIENCE FOR GLASS PRODUCTION

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CAPILLARY COUNTERACTION TO SEGREGATION OF PARTICLES IN GLASS BATCH PREPARATION

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Capillary-gravitational interaction between isomeric particles is analyzed with respect to segregation processes arising in glass batch preparation. The critical size of particles causing segregation is calculated for simple and complex capillary cells. The effect of the material wetting angle on the batch propensity for stratification is considered; the effect of mechanical overload and vibration in glass batch preparation is taken into account.

For appropriate control of the processes of material preparation, one should take into account capillary forces arising in moistening. In the general case, the force of capillary interaction between particles f_{cap} is made up of the Laplace pressure ΔP transmitted by the liquid via a section with surface area $\overline{\omega}$ and the surface tension force applied to the perimeter of the three-phase boundary C:

$$f_{cap} = f_{cap}^{v} + f_{cap}^{\omega} = \Delta P\,\overline{\omega} + \overline{\sigma} C, \qquad (1)$$

where $\overline{\sigma}$ is the projection of the surface tension vector applied to the three-phase boundary on the axis connecting solid particles.

Fischer was the first to use in 1926 the full equation for capillary forces when considering the attraction between two spheroid particles through a drop interlayer. By transforming Eq. (1) by means of geometric constructions he derived the following formula:

$$f_{cap} = \sigma\pi \sin\varphi\,[R\sin\varphi\,(1/\rho_2 - 1/\rho_1) + 2\sin(\varphi + \Theta)], \qquad (2)$$

where φ is half of the angle at the center upon which the cup rests (Fig. 1*a* – *c*); R is the spherical particle radius; ρ_1 and ρ_2 are the main radii of the meniscus curvature; Θ is the wetting angle of liquid wetting the surface of solid particles.

With a small cup volume ($\varphi \to 0$) Eq. (2) is transformed into the expression $f_{cap} = 2\sigma\pi R$, whereas when the capillary cell is completely filled with liquid ($\varphi \to 90°$), the force of interaction between two spheres is half as much.

Since the glass batch is a polydisperse system, let us write the parameters ρ_1 and ρ_2 taking into account the different sizes of spheroid particles. Neglecting the gravity effect, we obtain [1]

$$\rho_2 = \frac{R_1(1-\cos\varphi_1) + R_2(1-\cos\varphi_2) + l}{\cos(\varphi_1 + \Theta_1) + \cos(\varphi_2 + \Theta_2)}; \qquad (3)$$

$$\rho_1 = R_1 \sin\varphi_1 + \rho_2\,[\sin(\varphi_1 + \Theta_1) - 1], \qquad (4)$$

where l is the minimal distance between the surfaces of particles.

Formula (2) with the corresponding expression of the main curvature radii ρ_1 and ρ_2 from relationships (3) and (4)

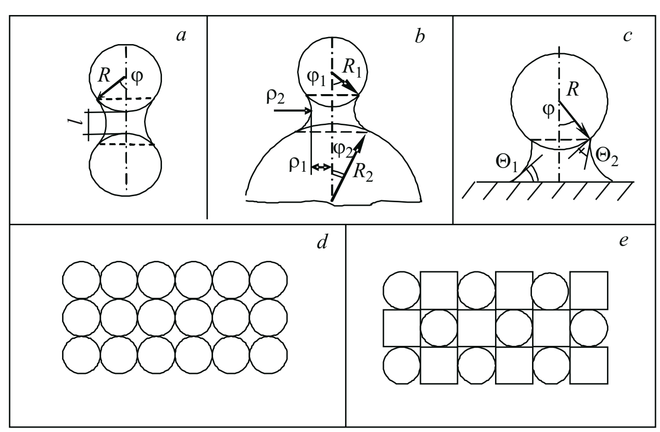

Fig. 1. Schemes of contacts of particles via liquid interlayer (*a* – *c*) and their respective capillary structure (*d*, *e*).

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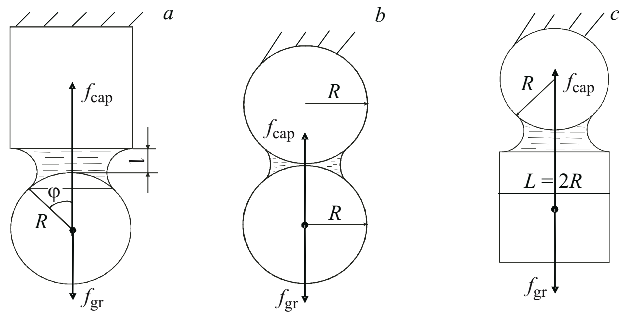


Fig. 2. Schemes of capillary contact: *a*) cube – sphere; *b*) sphere – sphere; *c*) sphere – cube.

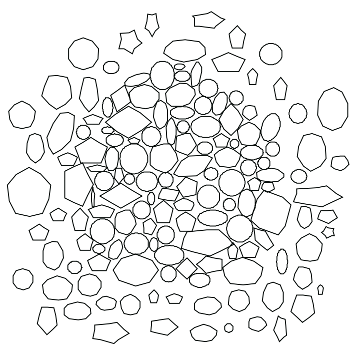


Fig. 3. Scheme of capillary-consolidated group of particles.

is the general formula for capillary adhesion of spheroid particles through a liquid interlayer. It encompasses the range of the ratio $R_1 : R_2$ varying from 1 to 0 (or to ∞). Its limiting cases are the contact between two identical spherical particles (Fig. 1*a*) and the adhesion of a sphere to a flat surface (Fig. 1*c*).

If the volume of liquid in the cup is small (φ_1 , $\varphi_2 < 20°$), then for $R_1 < R_2$ Eq. (2) becomes simplified [1]:

$$f_{cap} = \frac{2\sigma\pi R_1 (\cos\Theta_1 + \cos\Theta_2)}{1 + R_1 / R_2}.$$

Let us consider contacts with participation of isomeric particles for a small volume of capillary liquid. The limiting value of capillary force in the case of contact of two spheres with radius $R = R_1 = R_2$ amounts to:

$$f_{cap} = \sigma\pi R (\cos\Theta_1 + \cos\Theta_2)$$

and for contact of a sphere with a cubic particle $R_1 : R_2 \to 0$:

$$f_{cap} = 2\sigma\pi R (\cos\Theta_1 + \cos\Theta_2).$$

For simple cubic packing of isomeric particles the first case ($R_1 = R_2$) corresponds to a set of spherical particles (Fig. 1*d*) and the second case ($R_2 \gg R_1$) corresponds to a set of 50% spheres and 50% cubes (Fig. 1*e*) [1].

Capillary forces in disperse systems usually significantly exceed molecular, gravitational, magnetic, and other forces acting on solid particles. Therefore, we will analyze the possibility of segregation taking into account capillary interaction of particles and their gravity (Fig. 2). A simple variant of the analysis of capillary-gravitational interaction is described in [2]. Let us compare the force of adhesion of particles via a liquid cup with gravity force f_{gr}, using the above given capillary force:

$$f_{cap} = k\sigma\pi R (\cos\Theta_1 + \cos\Theta_2),$$

where k is a coefficient depending on the shape of the particles and the amount of liquid.

Thus, when a sphere adheres to a cube, the parameter k varies from 1 to 2 and in a sphere – sphere contact it varies from 0.5 to 1 where the maximum value corresponds to a vanishingly small volume of the cup.

The above described forms of capillary cells are particular cases, whereas in real conditions particles have different shapes (acute-angled, rounded, truncated, etc.). Figure 3 demonstrates the scheme of consolidation of glass batch particles after moistening.

The area of capillary-fixed particles is surrounded by dry contacts. A weak link is the bottom front of the conglomerate where individual particles may break off under the effect of gravity. This separation is opposed by the capillary force. Let us consider the possibility of particles breaking off in the static and dynamic states. Let us assume the upper particle to be fixed in the zone of the capillary-consolidated group. The subsequent analysis considers the lower particle, whose separation determines the segregation of the batch.

Figure 4 shows the effect of the gravitational field on the total adhesion force when the gravity and capillary forces are precisely opposite for three types of contacts (Fig. 2). For the contact "fixed cubic particle – spheroid particle" and for the contact of two spheroid particles the expression for the adhesion force takes the following form:

$$\Delta f = k\sigma\pi R (\cos\Theta_1 + \cos\Theta_2) - \frac{4}{3}\pi g d R^3 \qquad (5)$$

and for the contact "fixed spherical particle – cubic particle":

$$\Delta f = k\sigma\pi R (\cos\Theta_1 + \cos\Theta_2) - g d L^3,$$

where σ is the surface tension of the cup, J/m^2; d is the lower particle density, kg/m^3; g is the free fall acceleration, $kg \cdot m/sec^2$; L is the cubic particles size (Fig. 2*c*), m.

Under a mechanical impact on batch particles, which as a rule causes segregation, the real acceleration a differs from the free fall acceleration g. The load factor may amount to $3g$ and more. Accordingly, the pull force is more substantial as well.

It can be seen from Fig. 4 that the maximum adhesion force corresponds to the cubic particle – spherical particle

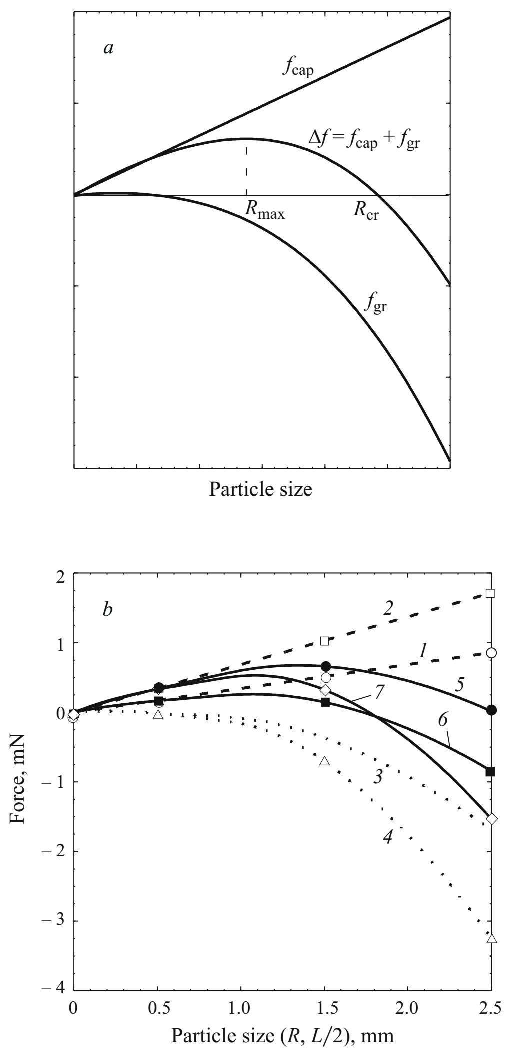


Fig. 4. Dependence of force acting on a particle stuck underneath on its size: *a*) general scheme of capillary-gravitational interaction; *b*) variation of total adhesion forces for different contacts; *1* and *2*) capillary force: sphere and cube; *3* and *4*) gravity: sphere and cube; *5*, *6*, and *7*) total force: cube – sphere; sphere – sphere, and sphere – cube, respectively.

contact. Let us find the critical size of a particle that can be retained by capillary forces developed by the cup under another particle, by means of equating the total force $\Delta f = 0$:

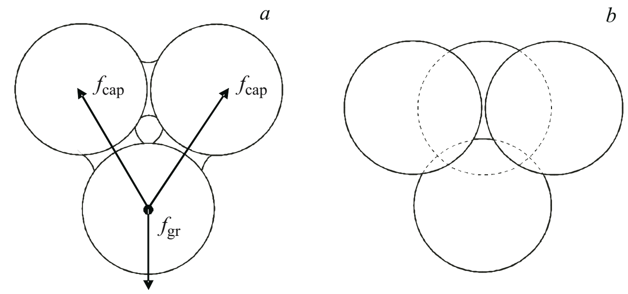
for the contact with a spheroid lower particle

$$R_{cr} = \sqrt{\frac{3k\sigma(\cos\Theta_1 + \cos\Theta_2)}{4dg}},$$

for the contact with a cubic lower particle

$$\frac{L_{cr}}{2} = \sqrt{\frac{k\sigma\pi(\cos\Theta_1 + \cos\Theta_2)}{8dg}},$$

where R_{cr} and L_{cr} are the critical sizes of the particles.



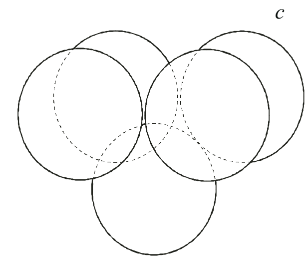


Fig. 5. Schemes of capillary contact: *a*) two spheres – one sphere; *b*) three sphere – one sphere; *c*) four spheres – one sphere.

By equating the derivative $d\Delta f/dR$ to zero, we find the size of the particle with the maximum adhesion force (Fig. 4*a*):

for the contact with the spheroid lower particle

$$R_{max} = \sqrt{\frac{k\sigma(\cos\Theta_1 + \cos\Theta_2)}{4dg}} = \frac{R_{cr}}{\sqrt{3}},$$

for the contact with the cubic lower particle

$$\frac{L_{max}}{2} = \sqrt{\frac{k\sigma\pi(\cos\Theta_1 + \cos\Theta_2)}{24dg}} = \frac{L_{cr}}{2\sqrt{3}}.$$

In real conditions different contacts of capillary-contracted particles are possible, in particular, more complex contacts formed by several particles (Fig. 5). Let us analyze the possibility of separation of a lower particle from a few upper ones.

The formula for calculating the total adhesion force for complex capillary cells will be similar to Eq. (5) with the additional factor k_n that takes into account the angle between the capillary force in pairwise contacts and the vertical axis and also the number of fixed particles. The factor k_n is calculated based on geometric constructions. For the contact "two spheres – one sphere" $k_n = \sqrt{3} \approx 1.732$, for "three spheres – one sphere" $k_n = 3\sqrt{2/3} \approx 2.449$, and for "four spheres – one sphere" $k_n = 2\sqrt{2} \approx 2.828$. Consequently, the "four spheres – one sphere" capillary contact is the most favorable for fixing the lower particle.

Let us consider the capillary-gravitational interaction of particles that have identical wettability (Fig. 6). According to our measurements, the water wetting angles Θ of sand, dolomite, and cullet particles are equal to 7 – 15°, and of graphite particles — 50 – 70°. As the wetting angle increases, the force of adhesion between the particles significantly grows,

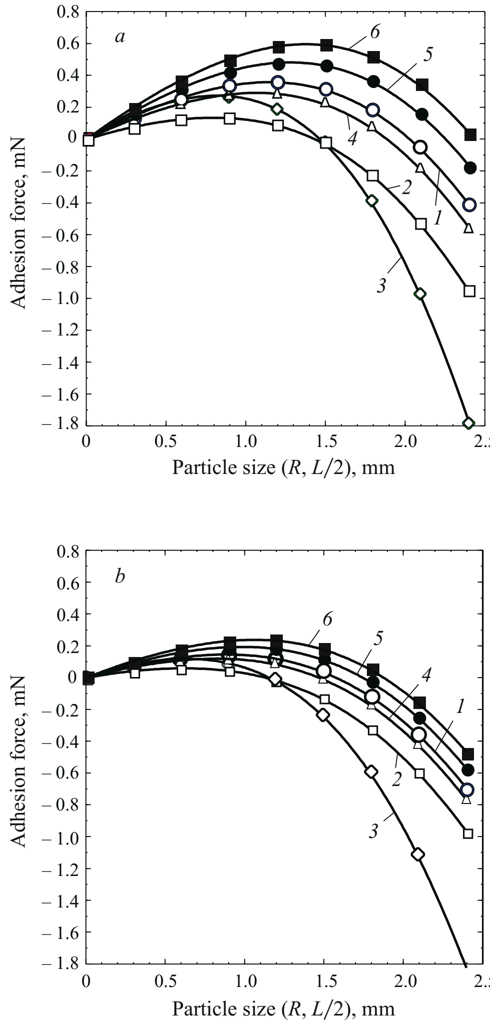


Fig. 6. Capillary-gravitational interaction of particles of different shapes: *a*) $\Theta_1 = \Theta_2 = 7°$; *b*) $\Theta_1 = \Theta_2 = 60°$; *1*) cube – sphere; *2*) sphere – sphere; *3*) sphere – cube; *4*) two spheres – one sphere; *5*) three spheres – one sphere; *6*) four spheres – one sphere.

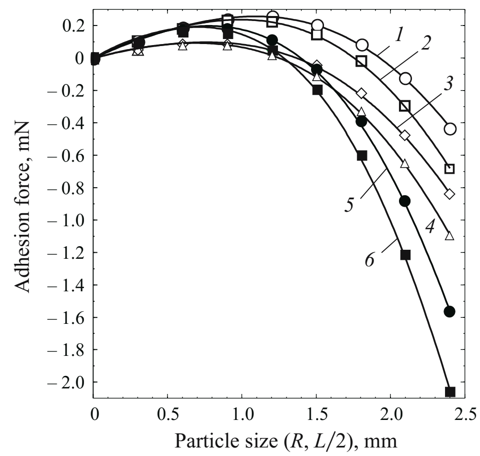


Fig. 7. Capillary-gravitational interaction of particles of different shapes with different wettability: *1*) $\Theta_1 = 7°$, $\Theta_2 = 60°$, cube – sphere; *2*) $\Theta_1 = 60°$, $\Theta_2 = 7°$, cube – sphere; *3*) $\Theta_1 = 7°$, $\Theta_2 = 60°$, sphere – sphere; *4*) $\Theta_1 = 60°$, $\Theta_2 = 7°$, sphere – sphere; *5*) $\Theta_1 = 7°$, $\Theta_2 = 60°$, sphere – cube; *6*) $\Theta_1 = 60°$, $\Theta_2 = 7°$, sphere – cube.

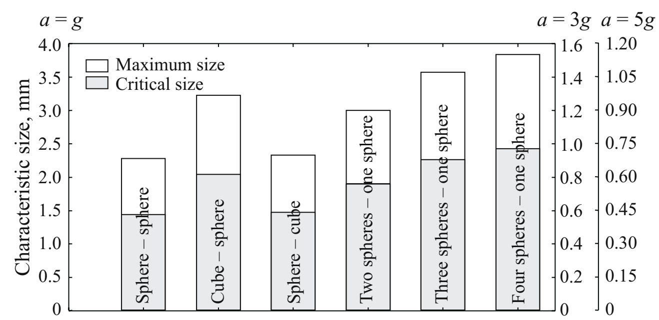


Fig. 8. Characteristic sizes of particles in different capillary cells (perfect wetting).

which is primarily related to the increasing main curvature radius ρ_2.

In a real batch structure, capillary contacts as a rule are formed by different particles that are taken into account by the wetting angle and density. Let us consider the strength of such capillary structure (Fig. 7).

Figures 6 and 7 show that the most intense increase in the total gravity force corresponds to the cubic particle.,

Particle sizes R_{cr}, L_{cr} and R_{max}, L_{max} for different capillary contacts and different acceleration values are shown in Fig. 8.

Let us assume that the acceleration a under mechanical overloads and vibration is equal to $3g$; then the particle size decreases approximately 1.73 times, whereas with $a = 5g$ it decrease 2.23 times. With a higher degree of moistening the calculated limiting parameters will be even smaller.

It can be noted that the size of particles, for which the possibility of segregation has been analyzed taking into account capillary and gravity forces, corresponds to the real sizes of glass batch particles, and the role of capillary forces is found significant even for relatively large particles.

Thus, the analysis of capillary-gravitational interaction of isometric particles as applied to segregation arising in glass batch preparation has shown that the capillary force is capable of counteracting the stratification of the batch.

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